Program : **B.Tech**

Subject Name: **Surveying**

Subject Code: **CE-303**

Semester: **3rd**

Unit 4

Curves: Classification and use; elements of circular curves, calculations, setting out curves by offsets and by theodolites, compound curves, reverse curves, transition curves, vertical curves, setting out.

- **Introduction**

Whenever the direction of a road or railway line is to be changed, curves are provided between the intersecting straights. This is necessary for smooth and safe movement of the vehicles and for the comfort of passengers. The curves required may be in the horizontal planes or in the vertical planes. Accordingly the curves are classified as horizontal curves and vertical curves. Curves are provided whenever a road changes its direction from right to left or changes its alignment from up to down (and vice versa). Curves are a critical element in the pavement design. They are provided with a maximum speed limit that should lie followed very strictly. Following the speed limit becomes essential as the exceed in speed may lead to the chances of the vehicle becoming out of control while negotiating a turn and thus increase the odds of fatal accidents. Also, it is very necessary that appropriate safety measures be adopted at all horizontal and vertical curves to make the infrastructure road user friendly and decrease the risks of hazardous circumstances. The low cost safety measures that can be adopted at curves included chevron signs, delineators, pavement markings, flexible posts, fluorescent strips, road safety barriers, rumble strips etc.

- **Types of Curves: -** There are two types of curves provided primarily for the comfort and ease of the motorists in the road namely:

1. **Horizontal Curve**
2. **Vertical Curve**

1. **Horizontal Curve: -** Horizontal curves are provided to change the direction or alignment of a road. Horizontal Curve is a circular curves or circular arcs. The sharpness of a curve increases as the radius is decrease which makes it risky and dangerous. The main design criterion of a horizontal curve is the provision of an adequate safe stopping sight distance.

- **Types of Horizontal Curve:**
  a) **Simple Curve:** A simple arc provided in the road to impose a curve between the two straight lines.
  b) **Compound Curve:** Combination of two simple curves combined together to curve in the same direction.
  c) **Reverse Curve:** Combination of two simple curves combined together to curve in the same direction.
  d) **Transition or Spiral Curve:** A curve that has a varying radius. It provided with a simple curve and between the simple curves in a compound curve. While turning a vehicle is

exposed to two forces. The first force which attracts the vehicle towards the ground is gravity. The second is centripetal force, which is an external force required to keep the vehicle on a curved path. At any velocity, the centripetal force would be greater for a tighter turn (smaller radius) than a broader one (larger radius). Thus, the vehicle would have to make a very wide circle in order to negotiate a turn. This issue is encountered when providing horizontal curves by designing roads that are tilted at a slight angle thus providing ease and comfort to the driver while turning. This phenomenon is defined as super elevation, which is the amount of rise seen on a given cross-section of a turning road, it is otherwise known as slope.

2. **Vertical Curve: -** Vertical curves are provided to change the slope in the road and may or may not be symmetrical. They are parabolic and not circular like horizontal curves. Identifying the proper grade and the safe passing sight distance is the main design criterion of the vertical curve, in crest vertical curve the length should be enough to provide safe stopping sight distance and in sag vertical curve the length is important as it influences the factors such as headlight sight distance, rider comfort and drainage requirements.

a) **Sag Curve** Sag Curves are those which change the alignment of the road from uphill to downhill.

b) **Crest Curve/Summit Curve** Crest Curves are those which change the alignment of the road from downhill to uphill. In designing crest vertical curves it is important that the grades be not] too high which makes it difficult for the motorists to travel upon it.

- **Designation of a curve** In India the sharpness of the curve is designated by the radius of the curve and by the degree of curvature. There are two different definitions of degree of curvature: **i) Arc Definition** **ii) Chord Definition**.

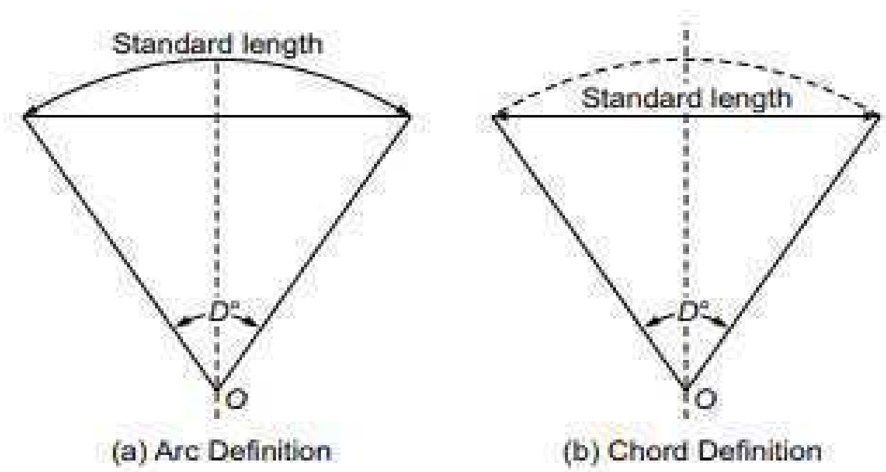


Fig: 2.4 Designation of curve

According to arc definition degree of curvature is defined as angle in degrees subtended by an arc of standard length this definition is generally used in highway practice. The length of standard arc used in FPS was 100 ft. In SI it is taken as 30 m. Some people take it as 20 m also.

According to chord definition degree of curvature is defined as angle in degrees subtended by a chord of standard length this definition is commonly used in railways. Earlier standard chord length used was 100 ft. Now in SI 30 m or 20 m is used as standard chord length.

Relationship between radius and degree of curve

(a) Arc Definition:

Let – R be the radius

– s be standard length

– D_a be degree of the curve

Referring to Fig. 2.4(a)

$$\therefore \quad s = R \times D_a \times \frac{\pi}{180}$$

$$\text{or} \quad R = \frac{s}{D_a} \times \frac{180}{\pi} \qquad \ldots(2.1)$$

If $s = 20$ m,

$$R = \frac{20}{D_a} \times \frac{180}{\pi} = \frac{1145.92}{D_a} \qquad \ldots(2.2a)$$

If $s = 30$ m,

$$R = \frac{30}{D_a} \times \frac{180}{\pi} = \frac{1718.87}{D_a} \qquad \ldots(2.2b)$$

(b) Chord Definition: Let D_c be degree of curve as per chord definition and s be the standard length of chord. Then referring to Fig. 2.4(b).

$$R \sin \frac{D_c}{2} = \frac{s}{2} \qquad \ldots(2.3)$$

When D_c is small, $\sin \frac{D_c}{2}$ may be taken approximately equal to $\frac{D_c}{2}$ radians.

Hence, for small degree curves (flat curves).

$$R \frac{D_c}{2} \times \frac{\pi}{180} = \frac{s}{2}$$

$$\text{or} \quad R = \frac{s}{D_c} \times \frac{180}{\pi} \qquad \ldots(2.4)$$

Comparing equations (2.1) and (2.4), we find for flat curves, arc definition and chord definitions give same degree of curve. As in railways flat curves are used, chord definition is preferred.

- **Elements of simple curves**

Terminologies in Simple Curve

- PC = Point of curvature. It is the beginning of curve.
- PT = Point of tangency. It is the end of curve.
- PI = Point of intersection of the tangents. Also called vertex
- T = Length of tangent from PC to PI and from PI to PT. It is known as sub tangent.
- R = Radius of simple curve, or simply radius.
- L = Length of chord from PC to PT. Point Q as shown below is the midpoint of L.
- L_c = Length of curve from PC to PT. Point M in the the figure is the midpoint of L_c.
- E = External distance, the nearest distance from PI to the curve.
- m = Middle ordinate, the distance from midpoint of curve to midpoint of chord.
- I = Deflection angle (also called angle of intersection and central angle). It is the angle of intersection of the tangents. The angle subtended by PC and PT at O is also equal to I, where O is the center of the circular curve from the above figure.
- x = offset distance from tangent to the curve. Note: x is perpendicular to T.
- θ = offset angle subtended at PC between PI and any point in the curve
- D = Degree of curve. It is the central angle subtended by a length of curve equal to one station. In English system, one station is equal to 100 ft and in SI, one station is equal to 20 m.
- Sub chord = chord distance between two adjacent full stations.

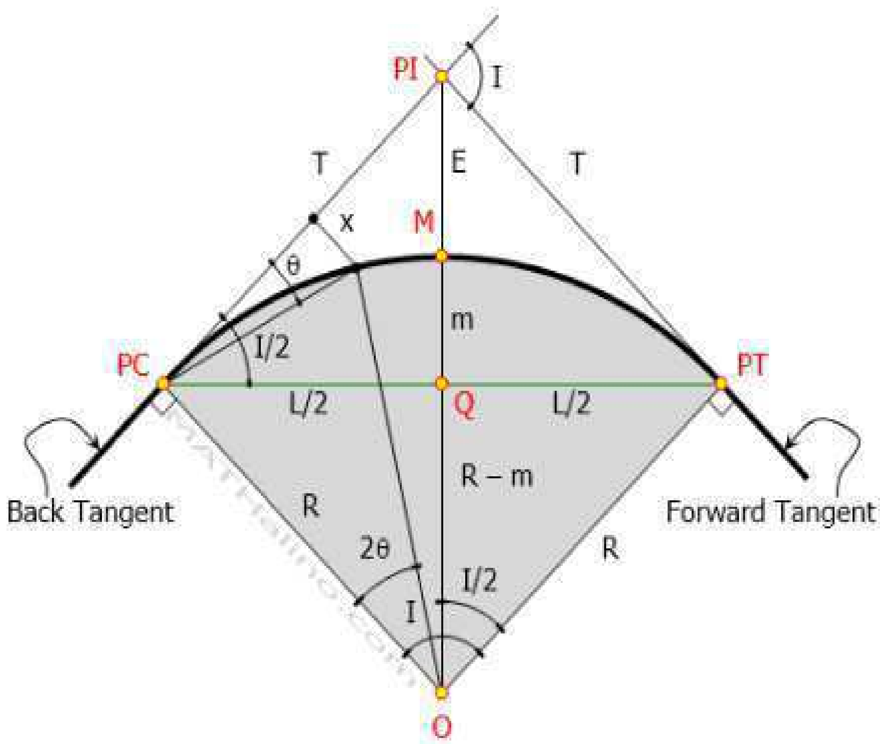


Fig: Simple Circular Curve

1. Length of Curve (l):

$$l = R\Delta, \text{ where } \Delta \text{ is in radians}$$
$$= R\Delta \times \frac{\pi}{180} \text{ if } \Delta \text{ is in degrees}$$

If the curve is designated by degree of curvature D_a for standard length of s, then

$$l = R\Delta \frac{\pi}{180}$$
$$= \frac{s}{D_a}\frac{180}{\pi} \cdot \Delta \frac{\pi}{180}, \quad \text{since from equation 2.1, } R = \frac{s}{D_a}\frac{180}{\pi}$$
$$l = \frac{s\Delta}{D_a} \quad \text{...(2.5)}$$

Thus,

If $\quad s = 30, \quad l = \dfrac{30\Delta}{D_a}$

and if $\quad s = 20 \text{ m}, \quad l = \dfrac{20\Delta}{D_a}$

2. Tangent Length (T):

$$T = T_1V = VT_2$$
$$= R \tan \frac{\Delta}{2} \quad \text{...(2.6)}$$

3. Length of Long Cord (L):

$$L = 2\ R \sin \frac{\Delta}{2} \quad \text{...(2.7)}$$

4. Mid-ordinate (M):

$$M = CD = CO - DO$$
$$= R - R\cos\frac{\Delta}{2}$$
$$= R\left(1 - \cos\frac{\Delta}{2}\right) = R \text{ Versin } \frac{\Delta}{2} \quad \text{...(2.8)}$$

5. External Distance (E):

$$E = VC = VO - CO$$

$$= R \sec\frac{\Delta}{2} - R$$
$$= R\left(\sec\frac{\Delta}{2} - 1\right) = R \text{ exsec } \frac{\Delta}{2}$$

Q.1 a circular curve has 300 m radius and 60° deflection angle. What is its degree by (a) arc definition and (b) chord definition of standard length 30 m? Also calculate (i) length of curve, (ii) tangent length, (iii) length of long chord,(iv) Mid-ordinate and (v) apex distance.

Solution

$$R = 300 \text{ m} \quad \Delta = 60^\circ$$

(a) Arc definition:

$$s = 30 \text{ m},$$

$$R = \frac{s}{D_a} \times \frac{180}{\pi}$$

$$\therefore \quad 300 = \frac{30 \times 180}{D_a \pi} \quad \text{or} \quad D_a = 5.730 \qquad \textbf{Ans.}$$

(b) Chord definition:

$$R \sin \frac{D_c}{2} = \frac{s}{2}$$

$$300 \sin \frac{D_c}{2} = \frac{30}{2}$$

$$\therefore \quad DC = 5.732 \qquad \textbf{Ans.}$$

(i) Length of the curve:

$$l = R\Delta \frac{\pi}{180} = 300 \times 60 \times \frac{\pi}{180} = 314.16 \text{ m} \qquad \textbf{Ans.}$$

(ii) Tangent length:

$$T = R \tan \frac{\Delta}{2} = 300 \tan \frac{60}{2} = 173.21 \text{ m} \qquad \textbf{Ans.}$$

(iii) Length of long chord:

$$L = 2 R \sin \frac{\Delta}{2} = 2 \times 300 \times \sin \frac{60}{2} = 300 \text{ m} \qquad \textbf{Ans.}$$

(iv) Mid-ordinate:

$$M = R\left(1 - \cos \frac{\Delta}{2}\right) = 300\left(1 - \cos \frac{60}{2}\right) = 40.19 \text{ m} \qquad \textbf{Ans.}$$

(v) Apex distance:

$$E = R\left(\sec \frac{\Delta}{2} - 1\right) = 300\left(\sec \frac{60}{2} - 1\right) = 46.41 \text{ m} \qquad \textbf{Ans.}$$

- **Setting Out a Simple Circular Curve**

After aligning the road/railway along *AA'*, when curve is to be inserted, alignment of *B¢B* is laid on the field by carefully going through the alignment map and field notes (Fig.)

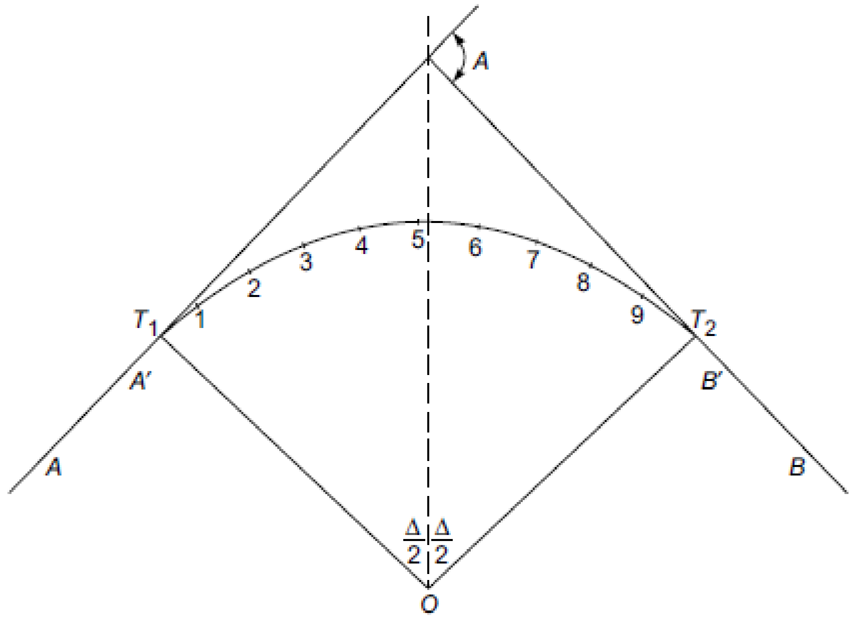


Fig: Setting out of a Circular Curve

By ranging from *AA'* and *BB'*, the vertex point *V* is determined. Setting a theodolite at *V*, the deflection angle is measured carefully. The tangent distance *T*1 is calculated. Subtracting this value from chainage of *V*, chainage of point of curve *T*1 is found. Adding length of curve to this chainage of *T*2 can be easily found.
Now pegs are to be fixed along the required curve at suitable intervals. It is impossible to measure along the curve. Hence, for fixing curve, chord lengths are taken as curved length. Chord length for peg interval is kep**t** 1/10th to 1/20th of radius of curve. When it is 1/10th of *R*, the error is 1 in 2500 and if it is 1/20th *R*, the error is 1 in 10,000. In practice the radius of the curve varies from 200 m to 1000 m. Hence, the chord length of 20 m is reasonably sufficient. For greater accuracy it may be taken as 10 m. In practice, pegs are fixed at full chain distances. For example, if 20 m chains used, chainage of *T*1 is 521.4 m and that of *T*2 is 695.8 m, the pegs are fixed at chainages 540, 560, 580 …, 660, 680 m. Thus, the chord length of first chord is 1.4 m while that of last one is 15.8 m. All intermediate chords are of 20 m. The first and last peg stations are known as sub-chord station while the others are full chord stations.
The various methods used for setting curves may be broadly classified as:
(i) Linear methods
(ii) Angular methods

- **LINEAR METHODS OF SETTING OUT SIMPLE CIRCULAR CURVES**

The following are some of the linear methods used for setting out simple circular curves:

a) Offsets from long chord

b) Successive bisection of chord

c) Offsets from the tangents–perpendicular or radial

d) Offsets from the chords produced.

- **Offsets from Long Chord**
  In this method, long chord is divided into an even number of equal parts. Taking centre of long chord as origin, for various values of *x*, the perpendicular offsets are calculated to the curve and the curve is set in the field by driving pegs at those offsets.

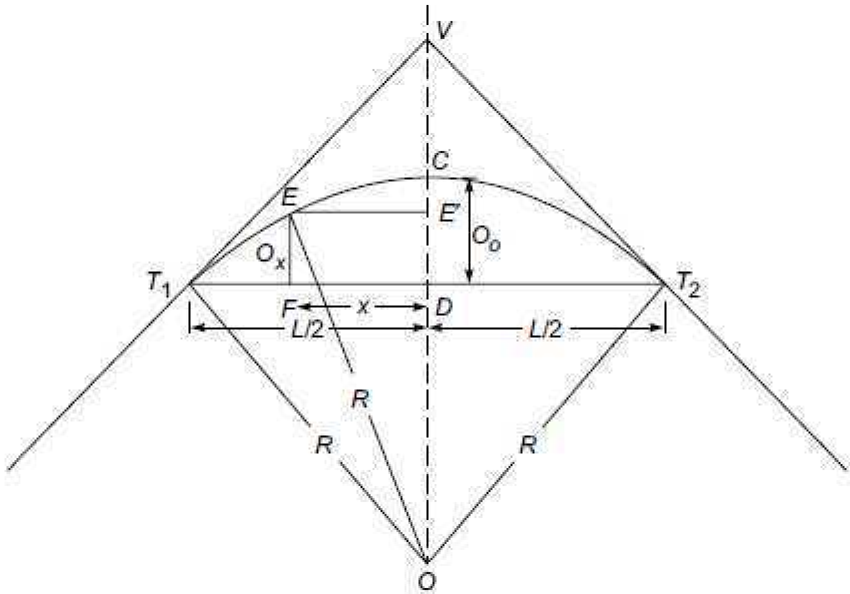


Fig: Setting out of curve by long chord method

R – radius of the curve

L – length of long chord

$^{0}0$ – mid-ordinate

O_x – ordinate at distance *x* from the mid-point of long chord

Ordinate at distance $x = O_x = E¢O – DO$

$$= \sqrt{R^2 - x^2} - \sqrt{R^2 - (L/2)^2} \quad ...(2.10)$$

The above expression holds good for *x*-values on either side of *D*, since *CD* is symmetric axis.

- **Successive Bisection of Chords**
  In this method, points on a curve are located by bisecting the chords and erecting the perpendiculars at the mid-point.

Perpendicular offset at middle of long chord (D) is

$$CD = R - R\cos\frac{\Delta}{2} = R\left(1 - \cos\frac{\Delta}{2}\right)$$

Let D_1 be the middle of T_1C. Then Perpendicular offset

$$C_1D_1 = R\left(1 - \cos\frac{\Delta}{4}\right)$$

Similarly,

$$C_2D_2 = R\left(1 - \cos\frac{\Delta}{8}\right)$$

Using symmetry points on either side may be set.

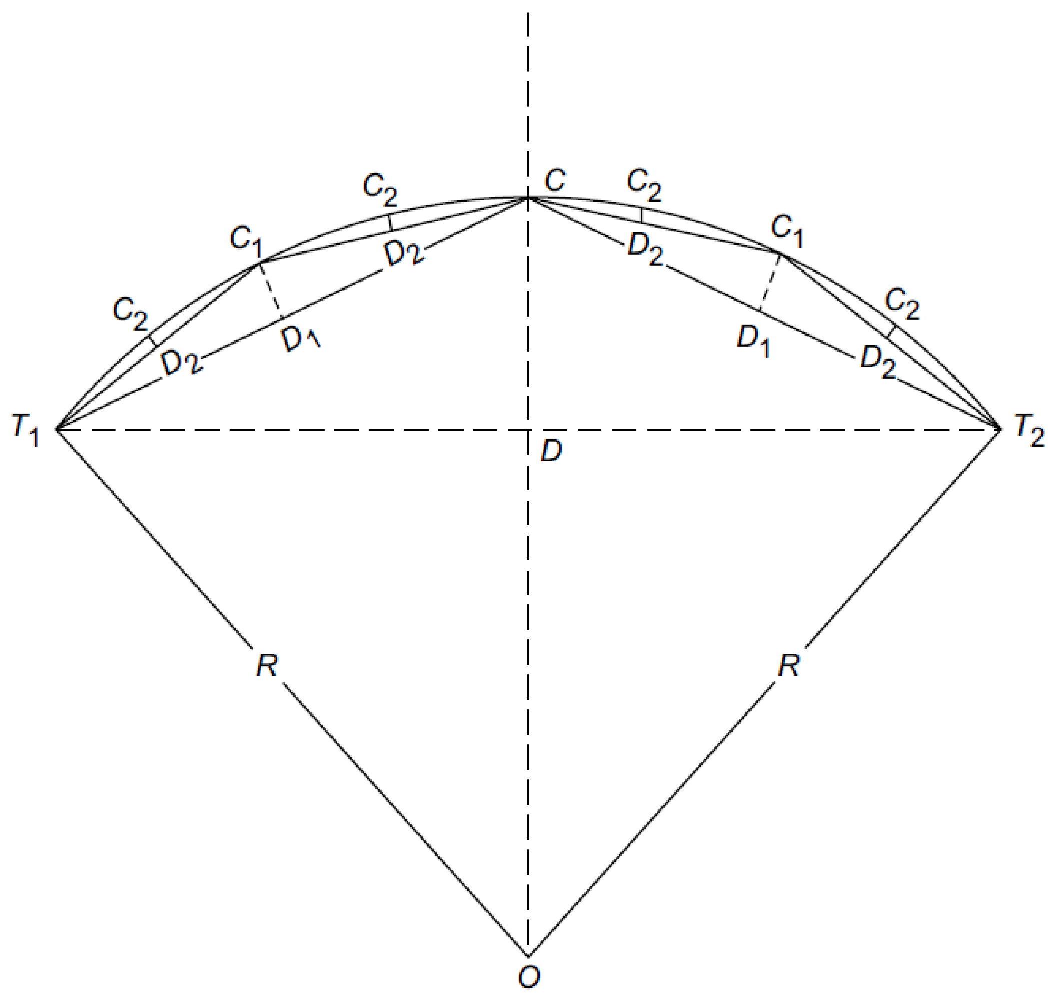


Fig: Offset of curve through Successive Bisection of Chords

- **Angular Method (Instrumental Method)**

The following are the angular methods which can be used for setting circular curves:

a) Rankine method of tangential (deflection) angles.

b) Two-theodolite method

c) Tacheometric method

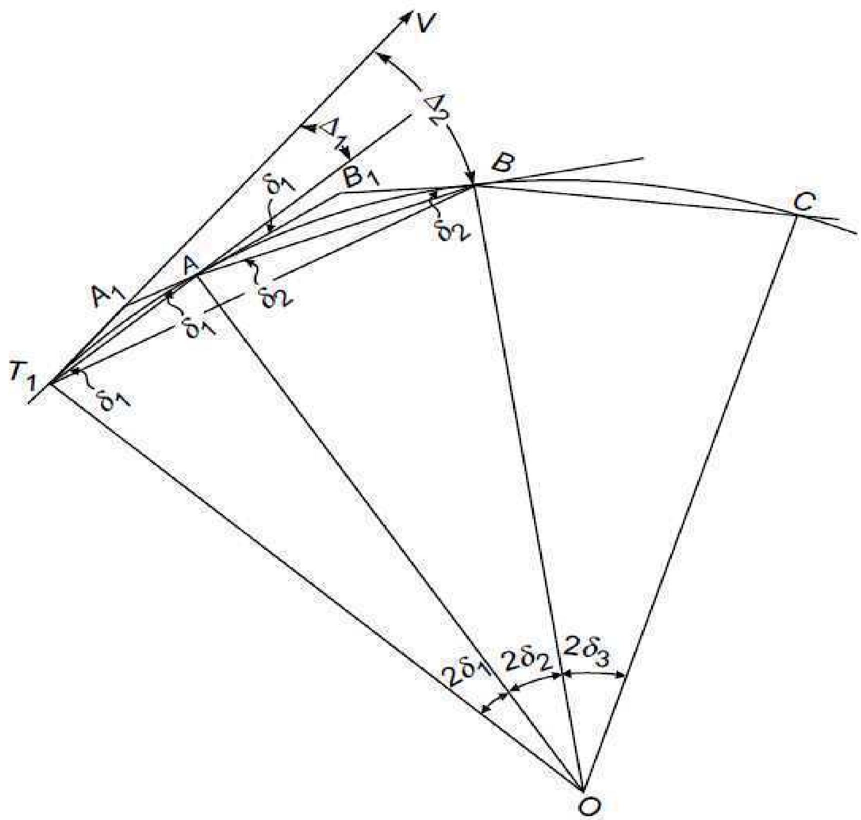
In these methods linear as well as angular measurements are used. Hence, the surveyor needs chain/tape and instruments to measure angles. Theodolite is the commonly used instrument. These methods are briefly explained in this chapter.

- **Rankine's Method of Tangential (or Deflection) Angles**

  A deflection angle to any point on the curve is the angle between the tangent at point of curve (*PC*) and the line joining that point to *PC* (D). Thus, referring to Fig. d_1 is the deflection angle of *A* and $d_1 + d_2$ is the deflection angle of *B*.

  In this method points on the curve are located by deflection angles and the chord lengths. The formula for calculating deflection angles of various chords can be derived as shown below:

Let *A*, *B*, *C* … be points on the curve. The chord lengths T_1A, *AB*, *BC*… be C_1, C_2, C_3… and d_1, d_2, d_3… tangential angles, which of the successive chords make with respective tangents. D_1, D_2, D_3… be deflection angles.



$$\angle VA_1A = \angle A_1T_1A + \angle A_1AT_1 = \delta_1 + \delta_1$$
$$= 2\delta_1$$

From the property of circular curve,

$$\angle T_1OA = \angle VA_1A = 2\delta_1$$

$$\therefore \qquad \text{Chord length} = C_1 = R \times 2\delta_1, \text{ if } \delta_1 \text{ is in radians}$$

$$= R \times 2\delta_1 \times \frac{\pi}{180}, \text{ if } \delta_1 \text{ is in degrees.}$$

$$\therefore \qquad \delta_1 = \frac{C_1}{2R} \times \frac{180}{\pi} \text{ degrees}$$

$$= \frac{C_1}{2R} \times \frac{180}{\pi} \times 60 \text{ minutes}$$

$$= 1718.87 \frac{C_1}{R} \text{ minutes}$$

Similarly,

$$\delta_2 = 1718.87 \frac{C_2}{R} \text{ minutes}$$

$$\Delta_1 = \text{Deflection angle of } AB = \delta_1$$

For the second chord

$$\Delta_2 = VT_1B = \Delta_1 + \delta_2 = \delta_1 + \delta_2$$

Similarly,

$$\Delta_n = \delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n = \Delta_{n-1} + \delta_n$$

Thus, the deflection angle of any chord is equal to the deflection angle for the previous chord plus the tangential angle of that chord. Note that if the degree of curve is *D* for standard length

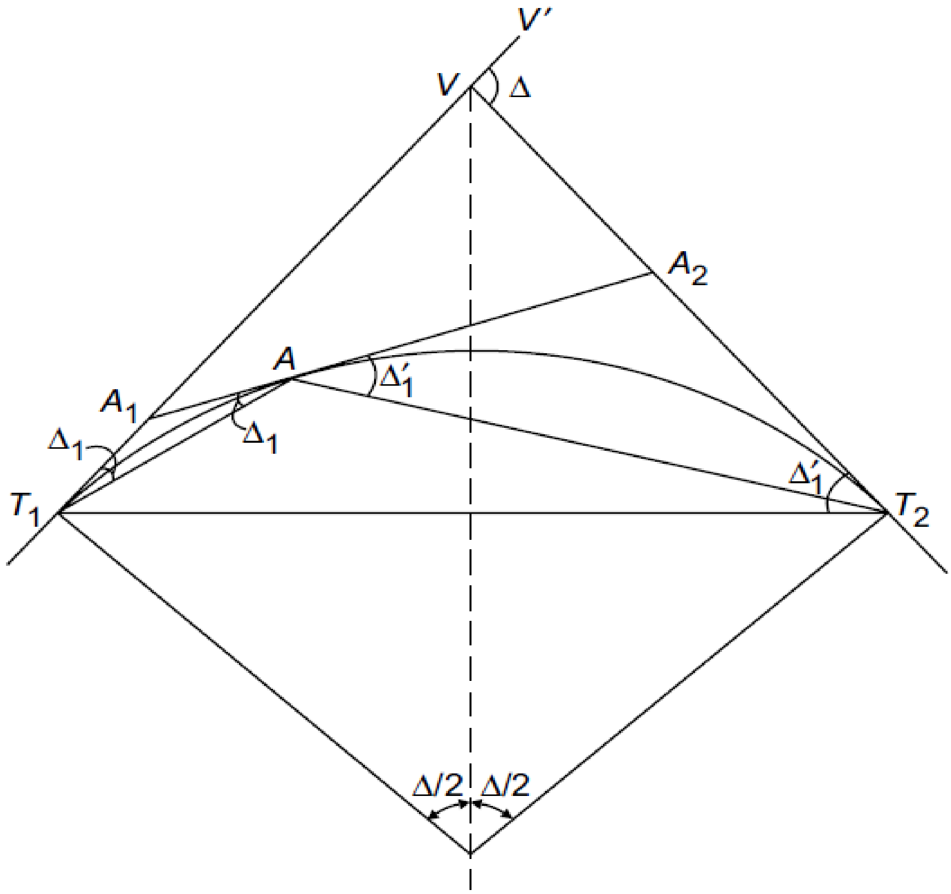
$$s = RD \times \frac{\pi}{180} \quad \text{or} \quad R = \frac{s}{D} \times \frac{180}{\pi}$$

If the degree of a curve is given, from equations (2.19) and (2.20) deflection angles can be found. Setting the theodolite at point of curve (*T*1), deflection angle D1 is set and chord length *C*1 is measured along this line to locate *A*. Then deflection angle D2 is set and *B* is located by setting *AB* = *C*2. The procedure is continued to lay the full curve.

- **Two-Theodolite Method**

In this method, two theodolites are used, one at the point of curve (*PC* i.e. at T_1) and another at the point of tangency (*PT* i.e. at T_2). For a point on the curve deflection angle with back tangent and forward tangent are calculated. The theodolites are set at *PC* and *PT* to read these angles and simultaneous ranging is made to get the point on the curve.

Let D_1 be deflection angle made by point *A* with back tangent and D¢$_1$ be the deflected angle made by the same point with forward tangent at T_2. The method of finding D_1 is already explained in the previous article. To find expression for D¢$_1$, draw a tangent at *A* intersecting back tangent at A_1 and forward tangent at A_2.
In triangle A_1T_1A, since A_1T_1 and A_1A both are tangents,



$$\angle A_1T_1A = \angle A_1AT_1 = \Delta_1$$

$\therefore$ Exterior angle $VA_1A_2 = 2\Delta_1$

Similarly, referring to triangle A_2AT_2, we get

Exterior angle $VA_2A_1 = 2\Delta'_1$

Now, considering the triangle VA_1A_2, the exterior angle

$$V'VA_2 = \angle VA_1A_2 + \angle VA_2A_1$$

i.e. $\Delta = 2\Delta_1 + 2\Delta'_1$

$\therefore$ $\Delta'_1 = \frac{\Delta}{2} - \Delta_1$

Hence, after finding the deflection angle with back tangent (D_1), the deflection angle D ¢1 with forward tangent can be determined.

Procedure to Set out Curve The following procedure is to be followed:

- Set the instrument at point of curve T_1, clamp horizontal plates at zero reading and sight V. Clamp the lower plate.
- Set another instrument at point of forward tangent T_2, clamp the horizontal plates at zero reading and sight V. Clamp the lower plate.

- Set horizontal angles D$_1$ and D¢$_1$ in the theodolites at T_1 and T_2 and locate intersecting point by ranging.
- Mark the point. Similarly fix other points.

- **Tacheometric Method**

If the terrain is rough, linear measurements may be replaced by the tacheometric measurements. The lengths of chord $T_1 A$, $T_1 B$ … may be calculated from the formula 2 R sin D$_1$, 2 R sin D$_2$ … etc. Then the respective staff intercepts s_1, s_2 …… may be calculated from the formula.

$$D = \frac{f}{i} s \cos^2 \theta + (f + d) \cos \theta$$
$$= ks \cos^2 \theta + C \cos \theta$$

- **Elements of the Compound Curve**

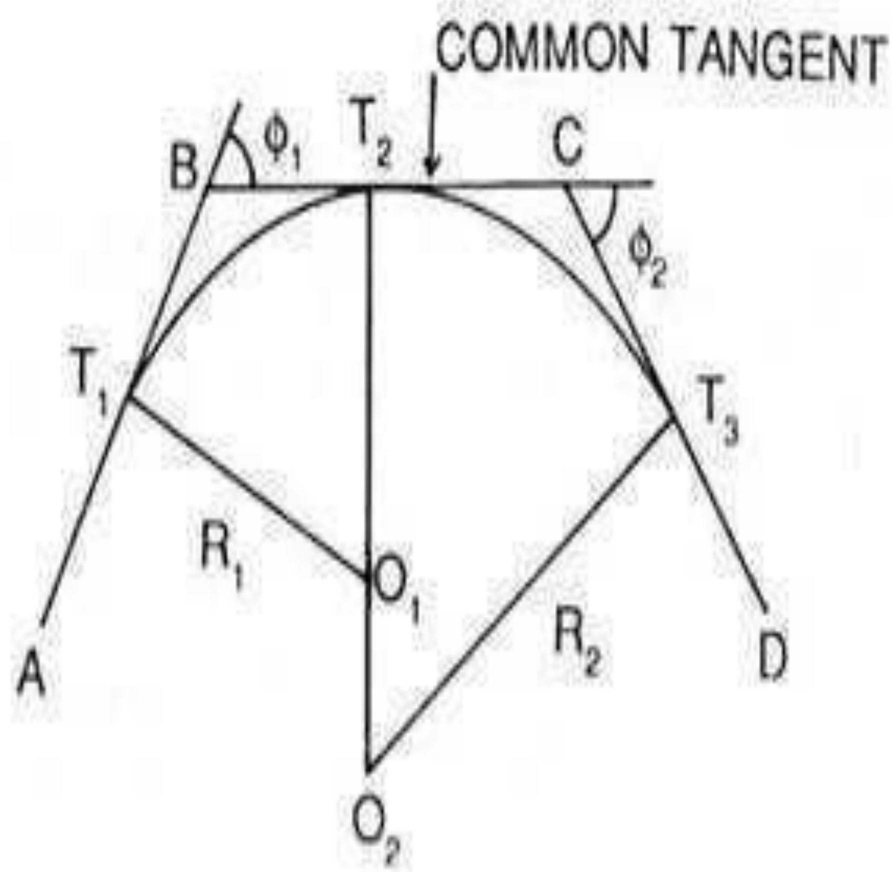


Fig: Compound Curve

From the property of circular curves.

$$\angle T_1O_1M = \angle MO_1C = \frac{\Delta_1}{2}$$

$$\angle CO_2N = \angle NO_2T_2 = \frac{\Delta_2}{2}$$

$$\angle VMC = \Delta_1 \quad \text{and} \quad \angle VNC = \Delta_2$$

$$\therefore \qquad \Delta = \Delta_1 + \Delta_2$$

$$t_1 = R_1 \tan\frac{\Delta_1}{2}$$

$$t_2 = R_2 \tan\frac{\Delta_2}{2}$$

$$\text{Length of common tangent} = MC + CN$$
$$= t_1 + t_2$$

i.e.
$$MN = R_1 \tan\frac{\Delta_1}{2} + R_2 \tan\frac{\Delta_2}{2}$$

From ΔVMN,

$$\frac{VM}{\sin\Delta_2} = \frac{VN}{\sin\Delta_1} = \frac{MN}{\sin[180-(\Delta_1+\Delta_2)]}$$

$$\therefore \qquad VM = \frac{\sin\Delta_2}{\sin(\Delta_1+\Delta_2)} MN$$

and
$$VN = \frac{\sin\Delta_1}{\sin(\Delta_1+\Delta_2)} MN$$

Now, $$TL_1 = t_1 + VM = t_1 + \frac{\sin\Delta_2}{\sin(\Delta_1+\Delta_2)}\left(R_1\tan\frac{\Delta_1}{2} + R_2\tan\frac{\Delta_2}{2}\right)$$

and $$TL_2 = t_2 + VN = t_2 + \frac{\sin\Delta_1}{\sin(\Delta_1+\Delta_2)}\left(R_1\tan\frac{\Delta_1}{2} + R_2\tan\frac{\Delta_2}{2}\right)$$

Of the seven quantities, R_s, R_L, T_s, T_L, D, D_1 and D_2, four must be known for setting the curve. The remaining three can be calculated from the equations.

- **Setting out compound curve involves the following steps:**

1. Knowing four quantities of the curve, calculate the remaining three quantities using equations.
2. Locate V, T_1 and T_2. Obtain the chainage of T_1 from the known chainage of V.
3. Calculate the length of the first arc and add it to the chainage of T_1 to obtain chainage of C. Similarly, compute the chainage of the second curve which when added to the chainage of C, gives the chainage of T_2.
4. ~~C~~alculate deflection angles for both the arcs.
5. Set the theodolite on T_1 and set out first arc as explained earlier.
6. Set the second curve from the deflection angle method.

- **Elements of a Reverse Curve**

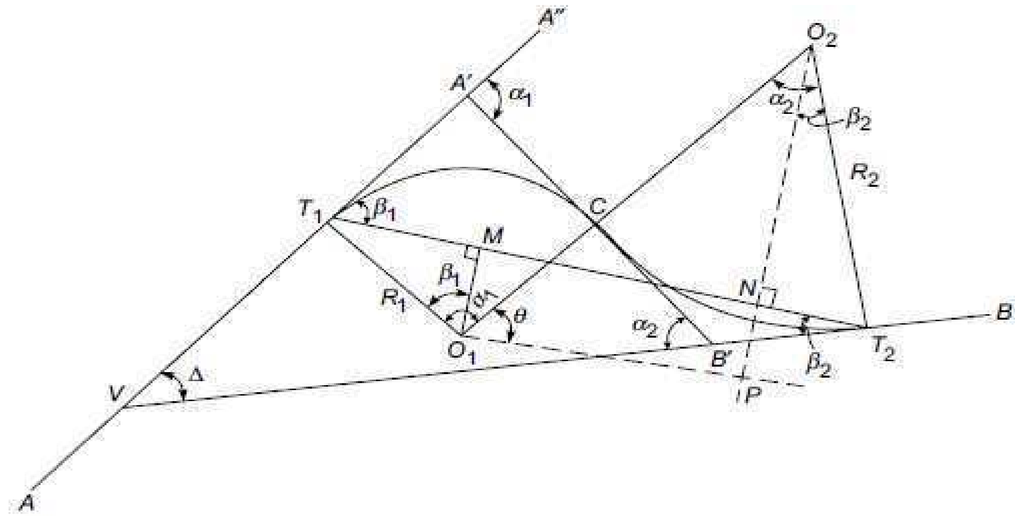


Fig: Reverse Curve

- **Elements of a Transition Curve** A curve of variable radius is termed as transition curve. It is generally provided on the sides of circular curve or between the tangent and circular curve and between two curves of compound curve or reverse curve etc. Its radius varies from infinity to the radius of provided for the circular curve. Transition curve helps gradual introduction of centrifugal force by gradual super elevation which provides comfort for the passengers in the vehicle without sudden jerking.

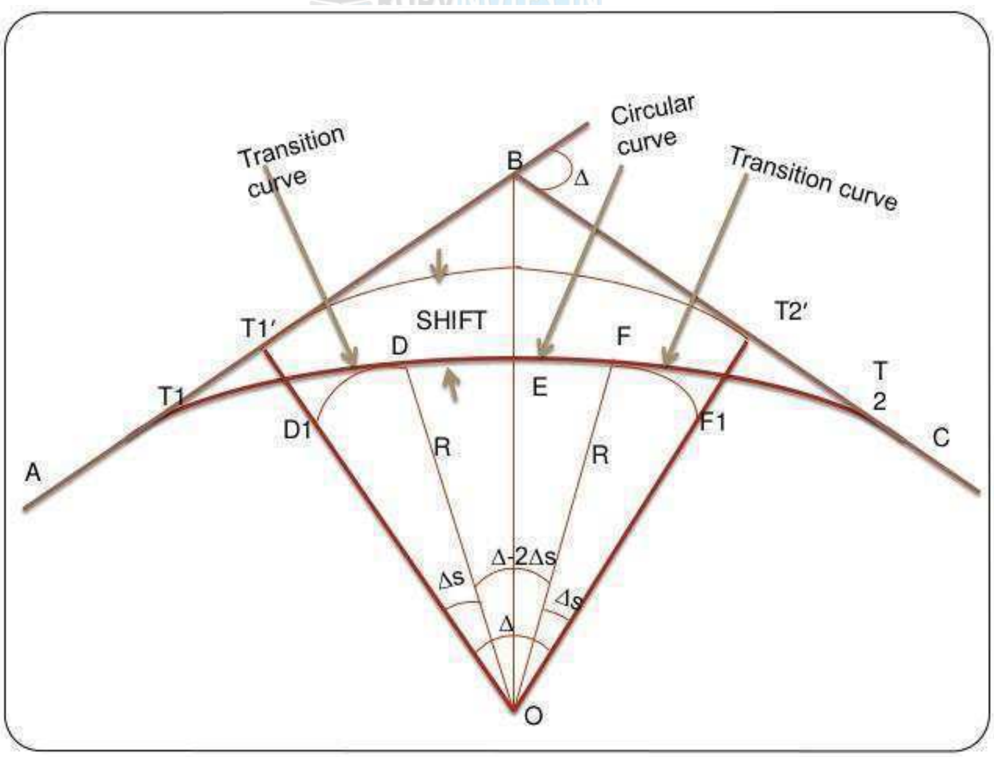


Fig: Transition Curve

